

DETERMINANTES- SOLUCIONES.

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16.-

$$\text{a) } |A| = \begin{vmatrix} 1 & 2 & 1 \\ 0 & 1 & 0 \\ 2 & 0 & 3 \end{vmatrix} = 1 \neq 0 \rightarrow \text{Existe } A^{-1} \quad A^{-1} = \frac{1}{|A|} (\text{Adj}(A))^t \quad \begin{pmatrix} 3 & -6 & -1 \\ 0 & 1 & 0 \\ -2 & 4 & 1 \end{pmatrix} = A^{-1}$$

$$|B| = \begin{vmatrix} 2 & 1 & 0 \\ 0 & 1 & 3 \\ 2 & 1 & 1 \end{vmatrix} = 2 \neq 0 \rightarrow \text{Existe } B^{-1} \quad B^{-1} = \frac{1}{|B|} (\text{Adj}(B))^t \quad \begin{pmatrix} -1 & -1/2 & 3/2 \\ 3 & 1 & -3 \\ -1 & 0 & 1 \end{pmatrix} = B^{-1}$$

$$\text{b) } AX = B \rightarrow A^{-1}AX = A^{-1}B \rightarrow X = A^{-1}B$$

$$X = A^{-1}B = \begin{pmatrix} 3 & -6 & -1 \\ 0 & 1 & 0 \\ -2 & 4 & 1 \end{pmatrix} \begin{pmatrix} 2 & 1 & 0 \\ 0 & 1 & 3 \\ 2 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 4 & -4 & -19 \\ 0 & 1 & 3 \\ -2 & 3 & 13 \end{pmatrix}$$

$$XB = A \rightarrow XBB^{-1} = AB^{-1} \rightarrow X = AB^{-1}$$

$$X = AB^{-1} = \begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 0 \\ 2 & 0 & 3 \end{pmatrix} \begin{pmatrix} -1 & -1/2 & 3/2 \\ 3 & 1 & -3 \\ -1 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 4 & 3/2 & -7/2 \\ 3 & 1 & -3 \\ -5 & -1 & 6 \end{pmatrix}$$

19.-

$$\text{a) } A(2I - A) = \begin{pmatrix} 2 & 1 & 1 \\ 2 & 3 & 2 \\ -3 & -3 & -2 \end{pmatrix} \left(2 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 2 & 1 & 1 \\ 2 & 3 & 2 \\ -3 & -3 & -2 \end{pmatrix} \right) = \begin{pmatrix} 2 & 1 & 1 \\ 2 & 3 & 2 \\ -3 & -3 & -2 \end{pmatrix} \begin{pmatrix} 0 & -1 & -1 \\ -2 & -1 & -2 \\ 3 & 3 & 4 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

b) $A(2I - A) = I \rightarrow A$ y $2I - A$ tienen inversa y cada una es la inversa de la otra.

$$A^{-1} = 2I - A$$

$$(2I - A)^{-1} = A$$

c) $k = 2$

20.-

$$|A| = t^2 + 4t - 12 = 0 \quad \text{para } t=2, t=-6 \quad . \text{ No tiene inversa.}$$

Calculamos A^{-1} para $t=1$:

$$A = \begin{pmatrix} 1 & 0 & 4 \\ 0 & 1 & 4 \\ -1 & 3 & 1 \end{pmatrix} \rightarrow |A| = -7$$

$$\frac{-1}{7} \begin{pmatrix} -11 & 12 & -4 \\ -4 & 5 & -4 \\ 1 & -3 & 1 \end{pmatrix} = A^{-1}$$

$$|B| = 1 - t^2 = 0 \begin{cases} t=1 \\ t=-1 \end{cases}$$

B no es invertible para $t=1$ ni para $t=-1$

$$B = \begin{pmatrix} 1 & 0 & 2 \\ 1 & 1 & 0 \\ 2 & 0 & 1 \end{pmatrix} \rightarrow |B| = -3$$

$$\frac{1}{3} \begin{pmatrix} -1 & 0 & 2 \\ 1 & 3 & -2 \\ 2 & 0 & -1 \end{pmatrix} = B^{-1}$$

22.-

$$AXB = \begin{pmatrix} 1 & 3 \\ 0 & -1 \end{pmatrix} \rightarrow X = A^{-1} \begin{pmatrix} 1 & 3 \\ 0 & -1 \end{pmatrix} B^{-1}; \quad X = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 3 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & -2 \end{pmatrix} = \begin{pmatrix} 5 & -8 \\ 2 & -3 \end{pmatrix}$$

24.-

$$(AB^t + C)X = D \rightarrow (AB^t + C)^{-1} (AB^t + C)X = (AB^t + C)^{-1}D \rightarrow X = (AB^t + C)^{-1}D$$

$$E = AB^t + C = \begin{pmatrix} -2 & 0 & 1 \\ 1 & -1 & 5 \end{pmatrix} \begin{pmatrix} 3 & 1 \\ 0 & 1 \\ -1 & 0 \end{pmatrix} + \begin{pmatrix} 1 & 2 \\ 3 & -1 \end{pmatrix} = \begin{pmatrix} -7 & -2 \\ -2 & 0 \end{pmatrix} + \begin{pmatrix} 1 & 2 \\ 3 & -1 \end{pmatrix} = \begin{pmatrix} -6 & 0 \\ 1 & -1 \end{pmatrix}$$

$$\frac{1}{6} \begin{pmatrix} -1 & 0 \\ -1 & -6 \end{pmatrix} = E^{-1} \quad X = (AB^t + C)^{-1}D = E^{-1}D \quad ; \quad \begin{pmatrix} 4/3 \\ 10/3 \end{pmatrix}$$

25)

$$\begin{pmatrix} -1 & 0 & 2 \\ -1 & 1 & 1 \\ 1 & 0 & -1 \end{pmatrix} = A^{-1} \quad X = \frac{1}{3} A^{-1} \cdot B$$

$$X = \frac{1}{3} \begin{pmatrix} -1 & 0 & 2 \\ -1 & 1 & 1 \\ 1 & 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 2 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix} = \frac{1}{3} \begin{pmatrix} 1 & 2 & 0 \\ 1 & 1 & 0 \\ 0 & -1 & 1 \end{pmatrix} = \begin{pmatrix} 1/3 & 2/3 & 0 \\ 1/3 & 1/3 & 0 \\ 0 & -1/3 & 1/3 \end{pmatrix}$$

